

12

TRIGONOMETRIC FUNCTIONS



12.4

Integration of Trigonometric Functions

Integrating Trigonometric Functions

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Each of the six equations pertaining to a rule of differentiation given in the last section may be integrated to yield a corresponding integration formula.

For example, integrating the equation $f'(x) = \cos x$ of Rule 1 with respect to x yields

$$\int \cos x \, dx = \int f'(x) \, dx = f(x) + C = \sin x + C$$

where C is a constant of integration.

Integrating Trigonometric Functions

The six rules of integration obtainable in this manner follow.

Trigonometric Integration Formulas

1. $\int \sin x \, dx = -\cos x + C$

2. $\int \cos x \, dx = \sin x + C$

3. $\int \sec^2 x \, dx = \tan x + C$

4. $\int \csc^2 x \, dx = -\cot x + C$

5. $\int \sec x \tan x \, dx = \sec x + C$

6. $\int \csc x \cot x \, dx = -\csc x + C$

Example 1

Find $\int \cos 3x \, dx$.

Solution:

Let's put $u = 3x$ so that $du = 3 \, dx$ and $dx = \frac{1}{3} du$.

Then

$$\int \cos 3x \, dx = \frac{1}{3} \int \cos u \, du$$

$$= \frac{1}{3} \sin u + C$$

$$= \frac{1}{3} \sin 3x + C$$

Use Rule 2.

Additional Trigonometric Integration Formulas

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The six integration formulas listed at the beginning of this section were immediate consequences of the six corresponding rules of differentiation.

We now complete the list of integration formulas for the basic trigonometric functions by giving the integration formulas for the tangent, secant, cosecant, and cotangent functions.

Additional Trigonometric Integration Formulas

$$7. \int \tan x \, dx = -\ln|\cos x| + C$$

$$8. \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$9. \int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$10. \int \cot x \, dx = \ln|\sin x| + C$$